Int J Theor Phys (2007) 46: 2868–2886
DOI 10.1007/s10773-007-9399-x

Quantum Field Formalism for the Electromagnetic Interaction of Composite Particles in a Nonrelativistic Gauge Model II

E.C. Manavella · R.R. Addad

Published online: 15 May 2007

Abstract By generalizing a model previously proposed, a classical nonrelativistic $U(1) \times U(1)$ gauge field model for the electromagnetic interaction of composite particles in $(2+1)$ dimensions is constructed. The model contains a Chern–Simons $U(1)$ field and the electromagnetic $U(1)$ field, and it describes both a composite boson system or a composite fermion one. The second case is considered explicitly. The model includes a topological mass term for the electromagnetic field and interaction terms between the gauge fields. By following the Dirac Hamiltonian formalism for constrained systems, the canonical quantization for the model is realized. By developing the path integral quantization method through the Faddeev–Senjanovic algorithm, the Feynman rules of the model are established and its diagrammatic structure is discussed. The Becchi–Rouet–Stora–Tyutin formalism is applied to the model. The obtained results are compared with the ones corresponding to the previous model.

Keywords Quantum field theory · Composite bosons and fermions

1 Introduction

As is well-known, the composite bosons [17, 18, 37, 38, 42, 43, 46, 49–52, 58, 59] and fermions [19–21, 23, 25–36, 39, 53] (CB, CF) theory occupies a preferential position in the description of the quantum Hall effect and its integer and fractional versions.

For this reason, we postulated [40] a classical nonrelativistic $U(1) \times U(1)$ gauge field model that describes the electromagnetic interaction of composite particles in $(2+1)$ di-

E.C. Manavella (✉)
Facultad de Ciencias Exactas, Ingeniería y Agrimensura, Universidad Nacional de Rosario, Av. Pellegrini 250, 2000 Rosario, Argentina
e-mail: manavell@ifir.edu.ar

E.C. Manavella · R.R. Addad
Instituto de Física Rosario, Universidad Nacional de Rosario, 27 de Febrero 210 bis, 2000 Rosario, Argentina

mensions. This model contains two $U(1)$ gauge fields, a Chern–Simons (CS) field a_μ [2, 57, 59] and the electromagnetic field A_μ.

Furthermore, we carried out the canonical quantization for the model. This was done by following the prescriptions of the Dirac Hamiltonian method for constrained systems [8, 9, 54].

Also, we developed the path integral quantization procedure by means of the Faddeev–Senjanovic (FS) algorithm [10, 48], used when the system has first- and second-class constraints. This enabled us to find the Feynman rules of the model.

Likewise, we studied a reduced version of the above model similar to one used within the framework of condensed matter [21].

On the other hand, we implemented the Becchi–Rouet–Stora–Tyutin (BRST) formalism to the model [4, 5, 15, 16, 22, 41, 54, 56]. We found that the generating functional obtained from this algorithm is equivalent to that found by following the FS method, as it should be expected.

In that paper, we considered explicitly the CF case.

As we suggested in that paper, an interesting point to take into account is to generalize the proposed model by adding different types of terms to the Lagrangian density.

In the present paper, we consider the following new terms:

(i) A topological mass term for the electromagnetic field.
(ii) Interaction terms between the CS and electromagnetic fields.

Furthermore, we analyze the resulting model following the same steps as the ones performed in [40] and we compare the obtained results with the ones corresponding to the original model.

We must point out that the present model does not try to be the most general model for composite particles within the framework of field theory. Merely, this model and the ones corresponding to Manavella [40] constitute generalizations of the model proposed by Halperin [21].

In this sense, in particular, we do not endow the CS field with topological mass, in order to preserve both the dynamics and the nature established for this field in [21].

The paper is organized as follows: In Sect. 2, we present our classical model and we perform its canonical quantization by means of the Dirac method. Afterwards, in Sect. 3, by developing the path integral quantization method through the FS algorithm, we establish the Feynman rules of the model and we analyze its diagrammatic structure. Next, in Sect. 4, we apply the BRST formalism to the model. Later on, in Sect. 5, we compare the obtained results with the ones corresponding to Manavella [40]. Finally, in Sect. 6, we display our conclusions and outlook.

2 Classical Model and Canonical Quantization

As we said, we had postulated [40] a classical nonrelativistic field model with $U(1) \times U(1)$ gauge symmetry for the electromagnetic interaction of composite particles in $(2+1)$ dimensions. We analyzed explicitly a CF system. We assumed that this system can be described by means of the following singular Lagrangian density:

$$\mathcal{L} = \mathcal{L}_{\mathrm{cf}}^{\mathrm{em}} + \mathcal{L}_{\mathrm{em}}, \tag{2.1}$$

where $\mathcal{L}_{\text{cf}}^{\text{em}}$ is written as

$$\mathcal{L}_{\text{cf}}^{\text{em}} = i\psi^\dagger \mathcal{D}_0 \psi + \frac{1}{2m_b}\psi^\dagger \vec{\mathcal{D}}^2 \psi - \mu\psi^\dagger\psi + \frac{1}{4\pi\tilde{\phi}}\varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \tag{2.2a}$$

and $\mathcal{L}_{\text{em}}$ reads

$$\mathcal{L}_{\text{em}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2.2b}$$

In (2.2), the Greek indices take the values $\mu, \nu, \rho = 0, 1, 2$.

We employed natural units where $\hbar = c = 1$. The Minkowskian metric used was $g_{\mu\nu} = \text{diag}(1, -1, -1)$ and $\varepsilon^{012} = \varepsilon^{12} = 1$.

In (2.2a), the covariant derivative, containing both the CS $U(1)$ gauge field a_μ and the electromagnetic $U(1)$ gauge field A_μ, is written as $\mathcal{D}_\mu = \partial_\mu - ia_\mu - ieA_\mu$ (the electron charge is taken as $-e$) and furthermore $\vec{\mathcal{D}}^2 = \mathcal{D}_1^2 + \mathcal{D}_2^2$. The matter field ψ is a charged spinorial field describing CF. m_b and μ are the band mass and the chemical potential of the electrons, respectively. $\tilde{\phi}$ is the strength of the flux tube, in units of the flux quantum 2π. (The fictitious charge of each particle that interacts with the fictitious gauge field has been chosen to have unit strength.)

In (2.2b), $F_{\mu\nu}$ is the electromagnetic field tensor.

By using the expression for the covariant derivative, we could rewrite (2.2a) as

$$\begin{aligned}\mathcal{L}_{\text{cf}}^{\text{em}} &= i\frac{\tau+1}{2}\psi^\dagger\,\partial_0\psi + i\frac{\tau-1}{2}\partial_0\psi^\dagger\psi + \psi^\dagger(a_0 + eA_0)\psi + \frac{1}{2m_b}\psi^\dagger\vec{\mathcal{D}}^2\psi - \mu\psi^\dagger\psi \\ &\quad + \frac{1}{4\pi\tilde{\phi}}\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho. \end{aligned}\tag{2.3}$$

In this equation, the kinetic fermionic term is written in the general form by using the arbitrary parameter τ [54].

On the other hand, it is known that the addition of a CS term to the Maxwell action leads to the topologically massive $(2+1)$-dimensional electrodynamics [24]. In that theory, a modified Gauss law appears with the result that any charged particle carries a magnetic flux proportional to its charge.

In the present paper, as it was proposed in the introduction, we consider a topological mass term for the electromagnetic field. Consequently, we use the electrodynamics above mentioned.

Besides, as we said, we introduce in the Lagrangian density interaction terms between the gauge fields.

So, we consider the following singular Lagrangian density, more general than the one given by (2.1):

$$\mathcal{L}_g = \mathcal{L}_{\text{cf}}^{\text{em}} + \mathcal{L}_{\text{tm}} + \mathcal{L}_{\text{int}}, \tag{2.4}$$

where $\mathcal{L}_{\text{cf}}^{\text{em}}$ is given by (2.3) and

$$\mathcal{L}_{\text{tm}} = \frac{1}{2\sigma}\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2.5a}$$

$$\mathcal{L}_{\text{int}} = \zeta\frac{e}{m_b}\varepsilon^{\mu\nu\rho}(a_\mu\partial_\nu A_\rho + A_\mu\partial_\nu a_\rho) + \eta\frac{e}{m_b}f_{\mu\nu}F^{\mu\nu}. \tag{2.5b}$$

The first term on the right-hand side of (2.5a) is the topological mass term for the electromagnetic field. The topological mass is given by $2\pi/\sigma$ and the real magnetic flux bound to the electrons is $e\sigma/2\pi$.

In (2.5b), $\mathcal{L}_{\text{int}}$ is the Lagrangian density corresponding to the interaction between the gauge fields and $f_{\mu\nu}$ is the CS field tensor. Furthermore, in that equation, the CS term is written in such a way to obtain symmetric expressions for the canonically conjugate momenta corresponding to the gauge fields.

Now, we are going to develop the Dirac canonical quantization procedure for the model.

The momenta $P^{\mathcal{I}} = (p^\mu, P^\nu, \pi^\dagger_\alpha, \pi_\beta)$ canonically conjugate to the independent dynamical field variables $A_{\mathcal{I}} = (a_\mu, A_\nu, \psi_\alpha, \psi^\dagger_\beta)$, respectively, are defined by $P^{\mathcal{I}} = \delta\mathcal{L}/\delta\dot{A}_{\mathcal{I}}$. In these equations, the compound index $\mathcal{I}$ runs over the components of the different field variables and the new Greek indices take the values $\alpha, \beta = 1, 2$.

The momenta have the following expressions:

$$p^0 = 0, \tag{2.6a}$$

$$p^i = \varepsilon^{ij}\left(\frac{1}{4\pi\tilde{\phi}}a_j + \zeta\frac{e}{m_b}A_j\right) + 2\eta\frac{e}{m_b}F^{0i}, \tag{2.6b}$$

$$P^0 = 0, \tag{2.6c}$$

$$P^i = \varepsilon^{ij}\left(\frac{1}{2\sigma}A_j + \zeta\frac{e}{m_b}a_j\right) + 2\eta\frac{e}{m_b}f^{0i} + F^{i0}, \tag{2.6d}$$

$$\pi^\dagger_\alpha = -i\frac{\tau+1}{2}\psi^\dagger_\alpha, \tag{2.6e}$$

$$\pi_\alpha = i\frac{\tau-1}{2}\psi_\alpha, \tag{2.6f}$$

where the Latin indices take the values $i, j = 1, 2$.

The nonvanishing fundamental equal-time ($x^0 = y^0$) Bose–Fermi brackets [6, 7] are given by

$$[a_\mu(x), p^\nu(y)]_- = \delta^\nu_\mu\delta(\vec{x}-\vec{y}), \tag{2.7a}$$

$$[A_\mu(x), P^\nu(y)]_- = \delta^\nu_\mu\delta(\vec{x}-\vec{y}), \tag{2.7b}$$

$$[\psi_\alpha(x), \pi^\dagger_\beta(y)]_+ = -\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.7c}$$

$$[\psi^\dagger_\alpha(x), \pi_\beta(y)]_+ = -\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.7d}$$

where we have used the notation $[.\,,.]_\mp$ to point out brackets between bosonic and fermionic Grassmann variables, respectively.

From (2.6), we find the following primary constraints:

$$\Phi^0_a = p^0 \approx 0, \tag{2.8a}$$

$$\Phi^0_A = P^0 \approx 0, \tag{2.8b}$$

$$\Omega^\dagger_\alpha = \pi^\dagger_\alpha + i\frac{\tau+1}{2}\psi^\dagger_\alpha \approx 0, \tag{2.8c}$$

$$\Omega_\alpha = \pi_\alpha - i\frac{\tau-1}{2}\psi_\alpha \approx 0, \tag{2.8d}$$

where $\alpha = 1, 2$.

The canonical Hamiltonian density is defined by $\mathcal{H}_c = \dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger \pi - \mathcal{L}$. By using (2.6), we have that

$$\begin{aligned}\mathcal{H}_c &= \frac{m_b}{2\eta e}\left(\frac{m_b}{4\eta e} p^i + P^i\right) p_i + \frac{m_b}{2\eta e}\left[\left(\frac{\zeta e}{m_b} + \frac{m_b}{8\pi\tilde{\phi}\eta e}\right) p_j + \frac{1}{4\pi\tilde{\phi}} P_j\right]\varepsilon^{ij} a_i \\ &\quad + \frac{m_b}{8\pi\tilde{\phi}\eta e}\left(\frac{\zeta e}{m_b} + \frac{m_b}{16\pi\tilde{\phi}\eta e}\right) a_i a^i + \partial_i a_0 p^i + \partial_i A_0 P^i - \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij} a_0 \partial_i a_j - \frac{\eta e}{m_b} f_{ij} F^{ij} \\ &\quad + \frac{1}{4} F_{ij} F^{ij} - \psi^\dagger (a_0 + e A_0)\psi - \frac{1}{2m_b}\psi^\dagger \vec{\mathcal{D}}^2 \psi + \frac{\zeta}{4\eta}\left(\frac{1}{\sigma} + \frac{\zeta}{2\eta}\right) A_i A^i \\ &\quad + \frac{\zeta}{2\eta}\varepsilon^{ij} A_i P_j + \frac{m_b}{4\eta e}\left(\frac{1}{\sigma} + \frac{\zeta}{\eta}\right)\varepsilon^{ij} A_i p_j - \frac{\zeta e}{m_b}\varepsilon^{ij} A_0 \partial_i a_j + \mu\psi^\dagger\psi \\ &\quad + \frac{m_b}{2\eta e}\left[\frac{\zeta}{8\pi\tilde{\phi}\eta} + \frac{1}{8\pi\tilde{\phi}\sigma} + \left(\frac{\zeta e}{m_b}\right)^2\right] a_i A^i - \frac{1}{2\sigma}\varepsilon^{ij} A_0 \partial_i A_j - \frac{\zeta e}{m_b}\varepsilon^{ij} a_0 \partial_i A_j. \end{aligned} \tag{2.9}$$

Furthermore, the primary Hamiltonian density is given by

$$\mathcal{H}_p = \mathcal{H}_c + \lambda_a \Phi_a^0 + \lambda_A \Phi_A^0 + \lambda_\alpha^\dagger \Omega_\alpha + \Omega_\alpha^\dagger \lambda_\alpha, \tag{2.10}$$

where λ_a and λ_A are bosonic Lagrange multipliers and $\lambda_\alpha^\dagger$ and λ_α are fermionic ones.

Now, we must impose the consistency condition on the primary constraints. Thus, we find the following secondary constraints:

$$\Phi_a^1 = [\Phi_a^0, H_p] = \partial_i p^i + \varepsilon^{ij}\left(\frac{1}{4\pi\tilde{\phi}}\partial_i a_j + \frac{\zeta e}{m_b}\partial_i A_j\right) + \psi^\dagger\psi \approx 0, \tag{2.11a}$$

$$\Phi_A^1 = [\Phi_A^0, H_p] = \partial_i P^i + \varepsilon^{ij}\left(\frac{1}{2\sigma}\partial_i A_j + \frac{\zeta e}{m_b}\partial_i a_j\right) + e\psi^\dagger\psi \approx 0, \tag{2.11b}$$

where $H_p = \int d^2x \mathcal{H}_p$ is the primary Hamiltonian.

It is easy to see that (2.11a), (2.11b) are the time components of the equations of motion corresponding to a_μ and A_μ, respectively.

When the consistency on the constraints (2.8c), (2.8d) is implemented, the Lagrange multipliers $\lambda_\alpha^\dagger$ and λ_α, $\alpha = 1, 2$, appearing in (2.10), remain determined, respectively. Furthermore, when the consistency on the constraints (2.11) is imposed, the obtained equations are satisfied automatically. Therefore, there are no further constraints.

Next, we find that the constraints (2.8a), (2.8b) are first-class whereas the constraints (2.8c), (2.8d) and (2.11) are second-class. However, these last ones do not constitute a minimal set of second-class constraints. This is due to the determinant of the matrix constructed with the Bose–Fermi brackets between these constraints vanishes.

So, there must be at least two linear combinations of second-class constraints which are independent of the above first-class constraints and these are also first-class. It is easy to see that there are only two of such combinations, which are

$$\begin{aligned}\Sigma_1 &= e\Phi_a^1 - \Phi_A^1 \\ &= e\left(\frac{1}{4\pi\tilde{\phi}} - \frac{\zeta}{m_b}\right)\varepsilon^{ij}\partial_i a_j + \left(\frac{\zeta e^2}{m_b} - \frac{1}{2\sigma}\right)\varepsilon^{ij}\partial_i A_j + e\partial_i p^i - \partial_i P^i \approx 0, \end{aligned} \tag{2.12a}$$

$$\Sigma_2 = \psi^\dagger \Omega - \psi \Omega^\dagger - \frac{i}{e}\Phi_A^1$$
$$= \psi^\dagger \pi - \psi \pi^\dagger - \frac{i}{e}\left(\frac{\zeta e}{m_b}\varepsilon^{ij}\partial_i a_j + \frac{1}{2\sigma}\varepsilon^{ij}\partial_i A_j + \partial_i P^i\right) \approx 0. \tag{2.12b}$$

Therefore, two second-class constraints can be eliminated and thus the final set of constraints is the following:

(i) The bosonic first-class constraints defined by the functions Σ_1, Σ_2, $\Sigma_3 = \Phi_a^0$, and $\Sigma_4 = \Phi_A^0$. As is well-known, these constraints are related to the symmetries of the gauge group $U(1) \times U(1)$ of the model.
(ii) The fermionic second-class constraints defined by $\Omega_\alpha^\dagger$ and Ω_α.

Now, we must go from the Bose–Fermi brackets to the Dirac ones $D(F)$ with regard to the matrix F constructed with the Bose–Fermi brackets between the second-class constraints. The $D(F)$ bracket between the functions $R(x)$ and $S(y)$ is defined by

$$[R(x), S(y)]^{D(F)} = [R(x), S(y)] - \int d^2u d^2v [R(x), \Gamma_I(u)] F_{IJ}^{-1}(\vec{u}, \vec{v}) [\Gamma_J(v), S(y)], \tag{2.13}$$

where F^{-1} is the inverse of the matrix F whose elements are $[\Gamma_I, \Gamma_J]$, $I, J = 1, \ldots, 4$, and $\Gamma_1 = \Omega_1^\dagger$, $\Gamma_2 = \Omega_2^\dagger$, $\Gamma_3 = \Omega_1$, and $\Gamma_4 = \Omega_2$ are the second-class constraints.

It is easy to see that the matrix F is given by

$$F = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \delta(\vec{x} - \vec{y}). \tag{2.14}$$

The determinant of F holds

$$\det F = \delta(\vec{x} - \vec{y}) \tag{2.15}$$

and its inverse reads

$$F^{-1} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \delta(\vec{x} - \vec{y}). \tag{2.16}$$

On the other hand, the extended Hamiltonian is defined as follows:

$$H_e = \int d^2x (\mathcal{H}_c + \rho^a \Sigma_a) - \int d^2x d^2y \Gamma_I(x) F_{IJ}^{-1}(\vec{x}, \vec{y}) [\Gamma_J(y), H_c], \tag{2.17}$$

where ρ^a, $a = 1, \ldots, 4$, are bosonic Lagrange multipliers.

Once we impose the $D(F)$ brackets, we must take the second-class constraints as equations strongly equal to zero. So, the second term on the right-hand side of (2.17) vanishes and then the extended Hamiltonian remains in the following way:

$$H_e = \int d^2x (\mathcal{H}_c + \rho^a \Sigma_a). \tag{2.18}$$

Furthermore, the following field variables remain determined:

$$\pi_\alpha^\dagger = -i\frac{\tau+1}{2}\psi_\alpha^\dagger, \tag{2.19a}$$

$$\pi_\alpha = i\frac{\tau-1}{2}\psi_\alpha. \tag{2.19b}$$

From (2.13), we find the following nonvanishing $D(F)$ brackets:
field-field:

$$[\psi_\alpha^\dagger(x), \psi_\beta(y)]_+^{D(F)} = -i\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.20a}$$

field-momentum:

$$[a_\mu(x), p^\nu(y)]_-^{D(F)} = \delta_\mu^\nu\delta(\vec{x}-\vec{y}), \tag{2.20b}$$

$$[A_\mu(x), P^\nu(y)]_-^{D(F)} = \delta_\mu^\nu\delta(\vec{x}-\vec{y}). \tag{2.20c}$$

Now, we must calculate the final Dirac brackets. For this, we must search for admissible gauge-fixing conditions $\Theta_a \approx 0$, one for each first-class constraint. These subsidiary conditions must verify that $\det([\Delta_A, \Delta_B]) \not\approx 0$, where $A, B = 1, \ldots, 8$ and $\Delta_a = \Sigma_a$, $\Delta_{4+a} = \Theta_a$, and must be compatible with the equations of motion.

We choose the following gauge-fixing conditions:

$$\Theta_1 = \partial^i a_i \approx 0, \tag{2.21a}$$

$$\Theta_2 = \partial^i A_i \approx 0, \tag{2.21b}$$

$$\Theta_3 = \nabla^2\left(A_0 - \frac{2\eta e}{m_b}a_0\right) + \varepsilon^{ij}\left(\frac{\zeta e}{m_b}\partial_i a_j + \frac{1}{2\sigma}\partial_i A_j\right) - \partial_i P^i \approx 0, \tag{2.21c}$$

$$\Theta_4 = \nabla^2 A_0 - \frac{m_b}{2\eta e}\left[\varepsilon^{ij}\left(\frac{1}{4\pi\tilde{\phi}}\partial_i a_j + \frac{\zeta e}{m_b}\partial_i A_j\right) - \partial_i p^i\right] \approx 0, \tag{2.21d}$$

which satisfy the above requirements.

The Dirac bracket between the functions $R(x)$ and $S(y)$ is defined by

$$\begin{aligned}[R(x), S(y)]^D &= [R(x), S(y)]^{D(F)} \\ &\quad - \int d^2u\, d^2v [R(x), \Delta_A(u)]^{D(F)} G_{AB}^{-1}(\vec{u}, \vec{v})[\Delta_B(v), S(y)]^{D(F)},\end{aligned} \tag{2.22}$$

where G^{-1} is the inverse of the matrix G whose elements are $[\Delta_A, \Delta_B]$, $A, B = 1, \ldots, 8$.

It is easy to show that the matrix G is written as follows:

$$G = \begin{pmatrix} 0 & 0 & 0 & 0 & e & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -ie^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & u & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -e & 0 & 0 & 0 & 0 & 0 & 0 & -u^{-1} \\ 1 & ie^{-1} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -u & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & u^{-1} & 0 & 0 & 0 \end{pmatrix} \nabla^2\delta(\vec{x}-\vec{y}), \tag{2.23}$$

where $u = 2\eta e/m_b$.

The determinant of G holds

$$\det G = -u^2(\nabla^2)^8\delta(\vec{x}-\vec{y}) \not\approx 0 \tag{2.24}$$

and its inverse matrix is written as

$$G^{-1} = \frac{1}{4\pi}\begin{pmatrix} 0 & 0 & e^{-1}u^{-2} & (eu)^{-1} & -e^{-1} & 0 & 0 & 0 \\ 0 & 0 & v & iu^{-1} & -i & -ie & 0 & 0 \\ -e^{-1}u^{-2} & -v & 0 & 0 & 0 & 0 & -u^{-1} & u^{-1} \\ -(eu)^{-1} & -iu^{-1} & 0 & 0 & 0 & 0 & 0 & 1 \\ e^{-1} & i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & ie & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & u^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -u^{-1} & -1 & 0 & 0 & 0 & 0 \end{pmatrix}\frac{1}{|\vec{x}-\vec{y}|}, \tag{2.25}$$

where $v = iu^{-2}(1+eu)$.

Once we impose the Dirac brackets, we must take the first-class constraints and the gauge-fixing conditions as equations strongly equal to zero. So, the second term on the right-hand side of (2.18) vanishes and then the extended Hamiltonian coincides with the canonical one. Moreover, the following field variables remain determined:

$$p^0 = 0, \tag{2.26a}$$

$$P^0 = 0, \tag{2.26b}$$

$$a_0(x) = \frac{m_b}{8\pi\eta e}\int d^2y \times \frac{[(\frac{\zeta e}{m_b} + \frac{m_b}{8\pi\tilde{\phi}\eta e})\varepsilon^{ij}\partial_i a_j + \frac{1}{2}(\frac{1}{\sigma}+\frac{\zeta}{\eta})\varepsilon^{ij}\partial_i A_j - \frac{m_b}{2\eta e}\partial_i p^i - \partial_i P^i](y)}{|\vec{x}-\vec{y}|}, \tag{2.26c}$$

$$A_0(x) = \frac{m_b}{8\pi\eta e}\int d^2y \frac{[\varepsilon^{ij}(\frac{1}{4\pi\tilde{\phi}}\partial_i a_j + \frac{\zeta e}{m_b}\partial_i A_j) - \partial_i p^i](y)}{|\vec{x}-\vec{y}|}. \tag{2.26d}$$

From (2.22), we obtain the following nonvanishing Dirac brackets:
field-field:

$$[\psi^\dagger_\alpha(x), \psi_\beta(y)]^D_+ = -i\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.27a}$$

field-momentum:

$$[a_i(x), p^j(y)]^D_- = [A_i(x), P^j(y)]^D_- = \delta^j_i\delta(\vec{x}-\vec{y}) - \frac{1}{4\pi}\partial^x_i\partial^{xj}\frac{1}{|\vec{x}-\vec{y}|}, \tag{2.27b}$$

momentum-momentum:

$$[p^1(x), p^2(y)]^D_- = -\frac{1}{4\pi\tilde{\phi}}\delta(\vec{x}-\vec{y}), \tag{2.27c}$$

$$[p^i(x), P^j(y)]^D_- = -\frac{\zeta e}{m_b}\varepsilon^{ij}\delta(\vec{x}-\vec{y}), \tag{2.27d}$$

$$[P^1(x), P^2(y)]^D_- = -\frac{1}{2\sigma}\delta(\vec{x}-\vec{y}). \tag{2.27e}$$

So, the dynamics of the classical model remains completely specified.

Finally, the canonical quantization is realized by replacing the Dirac brackets between field variables by the (anti) commutators between field operators according to the rule

$$[O_1(x), O_2(y)]^D \to -i[\hat{O}_1\hat{O}_2 - (-1)^{|O_1||O_2|}\hat{O}_2\hat{O}_1], \tag{2.28}$$

where $|O_i| = 0(1) (\mathrm{mod}\, 2)$ when O_i is a bosonic (fermionic) field variable, $i = 1, 2$.

On the other hand, let us note that a CB system can be treated similarly. In this case, the matter field is a charged scalar field. So, the fermionic second-class constraints (2.8c), (2.8d) turn into bosonic second-class ones. Furthermore, the fermionic brackets (2.7c), (2.7d), (2.20a), and (2.27a) turn into bosonic ones. Thus, after (2.28) is imposed, the Dirac brackets (2.27a) become commutators.

3 Path Integral Quantization, Feynman Rules, and Diagrammatic Structure

We develop the Feynman path integral quantization method according to the FS formalism due to the fact that the model has first- and second-class constraints. So, we write the generating functional for the model by means of the following path integral:

$$Z = \int \mathbb{D}a_\mu \mathbb{D}p^\mu \mathbb{D}A_\nu \mathbb{D}P^\nu \mathbb{D}\psi_\alpha \mathbb{D}\pi^\dagger_\alpha \mathbb{D}\psi^\dagger_\beta \mathbb{D}\pi_\beta \delta(\Gamma_I)(\det F)^{1/2}\delta(\Delta_A)(\det G)^{1/2}$$
$$\times \exp\left[i\int d^3x(\dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger\pi - \mathcal{H}_e)\right], \tag{3.1}$$

where $\delta(\Gamma_I)$ and $\delta(\Delta_A)$ are products of Dirac delta functions and the Hamiltonian density $\mathcal{H}_e$ was given in (2.17).

The determinant of the matrix F does not depend neither on the independent dynamical field variables nor on the corresponding canonically conjugate momenta (see (2.15)). Thus, we include $(\det F)^{1/2}$ in the path integral normalization factor. The same occurs with the other determinant appearing in (3.1) (see (2.24)).

By means of the Dirac deltas $\delta(\Delta_3)$, $\delta(\Delta_4)$, $\delta(\Gamma_1)$, $\delta(\Gamma_2)$, $\delta(\Gamma_3)$, and $\delta(\Gamma_4)$, we calculate the path integrals over p^0, P^0, $\pi^\dagger_1$, $\pi^\dagger_2$, π_1, and π_2, respectively.

Moreover, we can write $\delta(\Delta_7) = \delta(a_0 - \frac{m_b}{2\eta e}A_0 - f)$ and $\delta(\Delta_8) = \delta(A_0 - g)$, where

$$f(x) = \frac{m_b}{8\pi\eta e}\int d^2y \frac{(\frac{\zeta e}{m_b}\varepsilon^{ij}\partial_i a_j + \frac{1}{2\sigma}\varepsilon^{ij}\partial_i A_j - \partial_i P^i)(y)}{|\vec{x} - \vec{y}|}, \tag{3.2a}$$

$$g(x) = \frac{m_b}{8\pi\eta e}\int d^2y \frac{(\frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}\partial_i a_j + \frac{\zeta e}{m_b}\varepsilon^{ij}\partial_i A_j - \partial_i p^i)(y)}{|\vec{x} - \vec{y}|}, \tag{3.2b}$$

and so we make the integrations over a_0 and A_0.

On the other hand, by using the Fourier integral corresponding to the Dirac delta, we have that $\delta(\Delta_n) = \int \mathbb{D}\Lambda_n \exp(i\int d^3x\, \Lambda_n\Delta_n)$, $n = 1, 2$.

Consequently, the generating functional takes the form

$$Z = \int \mathbb{D}a_i \mathbb{D}p^i \mathbb{D}A_j \mathbb{D}P^j \mathbb{D}\psi_\alpha \mathbb{D}\psi^\dagger_\beta \mathbb{D}\Lambda_n \delta(\partial^l a_l)\delta(\partial^m A_m)\exp\left(i\int d^3x \mathcal{L}^*\right), \tag{3.3}$$

where

$$\mathcal{L}^* = \dot{a}_i p^i + \dot{A}_i P^i + \frac{i}{2}[(\tau - 1)\dot{\psi}^\dagger\psi - (\tau + 1)\dot{\psi}\psi^\dagger] - \mathcal{H}^*, \tag{3.4}$$

with

$$\mathcal{H}^* = \mathcal{H}_e^* - \Lambda_n \Delta_n, \tag{3.5}$$

where $\mathcal{H}_e^*$ is the original $\mathcal{H}_e$ subject to the integrations that we have just done.

Due to the arbitrariness of the Lagrange multipliers Λ_n in (3.5), it is possible to make a scale change in the corresponding integration variables in such a way that $\mathcal{H}^* = \mathcal{H}_c$ [54].

As it is can be seen, the path integrals over p^i and P^j are Gaussian. Therefore, these integrals can also be calculated.

So, the generating functional remains

$$Z = \int \mathbb{D}a_\mu \mathbb{D}A_\nu \mathbb{D}\psi_\alpha \mathbb{D}\psi_\beta^\dagger \delta(\partial^l a_l)\delta(\partial^m A_m) \exp\left(i \int d^3x \mathcal{L}_g\right), \tag{3.6}$$

where $\mathcal{L}_g$ is the starting Lagrangian density given by (2.4).

This result was expected. Nevertheless, we think necessary to start formally from the canonical path integral (3.1) and to show that it is possible to arrive at the Lagrangian path integral (3.6). This is because, as is well-known, there are many field theories in which the simple Lagrangian path integral can not be obtained from the canonical one (see, for instance, [47] and references therein).

Finally, we use the Faddeev–Popov trick. So, we write the gauge-fixing conditions in the form $\partial^\mu a_\mu(x) = c_a(x)$ and $\partial^\mu A_\mu(x) = c_A(x)$ (generalized Lorentz gauge). By considering the first of these conditions, we write $\delta[\partial^\mu a_\mu(x) - c_a(x)] = \int \mathbb{D}c_a(x) \exp\{i \frac{\lambda_a}{2} \times \int d^3x [\partial^\mu a_\mu(x)]^2\}$, with a Gaussian weight independent of $c_a(x)$. Therefore, the integrand in (3.6) does not depend on $c_a(x)$ and the integration over this quantity can be performed, appearing $\exp\{i \frac{\lambda_a}{2} \int d^3x [\partial^\mu a_\mu(x)]^2\}$ instead of $\delta[\partial^\mu a_\mu(x) - c_a(x)]$. We proceed analogously for the second condition.

In this way, the generating functional remains

$$Z = \int \mathbb{D}a_\mu \mathbb{D}A_\nu \mathbb{D}\psi_\alpha \mathbb{D}\psi_\beta^\dagger \exp\left(i \int d^3x \mathcal{L}_{\text{eff}}\right), \tag{3.7}$$

where the Lagrangian density $\mathcal{L}_{\text{eff}}$ is expressed in terms of the independent dynamical field variables, a_μ, A_ν, ψ_α, and $\psi_\beta^\dagger$, and so constitutes the effective Lagrangian density of the model. This is given by

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_g + \mathcal{L}_{\text{fix}}, \tag{3.8}$$

where

$$\mathcal{L}_{\text{fix}} = \frac{\lambda_a}{2}(\partial^\mu a_\mu)^2 + \frac{\lambda_A}{2}(\partial^\mu A_\mu)^2. \tag{3.9}$$

Now, we are going to establish the Feynman rules of the model [55].

Since the interaction terms (2.5b) are quadratic, these must contribute to the propagators. Therefore, the only possibility is to consider a unique auxiliary extended quantity $X_\Lambda = (a_\mu, A_\nu)$, where the compound index Λ runs over the components of the field variables [14]. In this way, the Lagrangian density (3.8), written in terms of this quantity, can be partitioned as follows:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}(X_\Lambda) + \mathcal{L}_{\text{eff}}(\psi, \psi^\dagger) + \mathcal{L}_{\text{eff}}^{\text{int}}(X_\Lambda, \psi, \psi^\dagger), \tag{3.10}$$

where

$$\mathcal{L}_{\text{eff}}(X_\Lambda) = \frac{1}{2} X_\Lambda (D^{-1})^{\Lambda\Pi} X_\Pi, \tag{3.11a}$$

$$\mathcal{L}_{\text{eff}}(\psi, \psi^\dagger) = \psi^\dagger G^{-1} \psi, \tag{3.11b}$$

$$\mathcal{L}_{\text{eff}}^{\text{int}}(X_\Lambda, \psi, \psi^\dagger) = \psi^\dagger V_\Lambda X^\Lambda \psi + \psi^\dagger X_\Lambda W^{\Lambda\Pi} X_\Pi \psi. \tag{3.11c}$$

The matrix D^{-1} appearing in (3.11a) is nondegenerate. As a result, the propagator $D_{\Lambda\Pi}(k)$ of the gauge field X_Λ, in the momentum space, can be obtained. This is given by

$$D_{\Lambda\Pi}(k) = \begin{pmatrix} M_{\mu\nu}(k) & L_{\mu\nu}(k) \\ L_{\mu\nu}(k) & N_{\mu\nu}(k) \end{pmatrix}, \tag{3.12}$$

where

$$M_{\mu\nu}(k) = \mu_1(k^2) g_{\mu\nu} + \mu_2(k^2) k_\mu k_\nu - i\mu_3(k^2)\varepsilon_{\mu\nu\rho} k^\rho, \tag{3.13a}$$

$$L_{\mu\nu}(k) = \lambda_1(k^2) g_{\mu\nu} + \lambda_2(k^2) k_\mu k_\nu - i\lambda_3(k^2)\varepsilon_{\mu\nu\rho} k^\rho, \tag{3.13b}$$

$$N_{\mu\nu}(k) = \nu_1(k^2) g_{\mu\nu} + \nu_2(k^2) k_\mu k_\nu - i\nu_3(k^2)\varepsilon_{\mu\nu\rho} k^\rho. \tag{3.13c}$$

In (3.13), the coefficients are given by

$$\mu_1(k^2) = \frac{(4a^2 - k^2)[c(4ad + c) + d^2k^2]}{\alpha(k^2)}, \tag{3.14a}$$

$$\mu_2(k^2) = \frac{1}{\lambda_a} \frac{\beta(k^2)}{k^4 \alpha(k^2)}, \tag{3.14b}$$

$$\mu_3(k^2) = \frac{1}{2} \frac{(4a^2 - k^2)[4ae + (4ad^2 + g)k^2]}{k^2 \alpha(k^2)}, \tag{3.14c}$$

$$\lambda_1(k^2) = \frac{-4a^2[2d(ab + c^2) + bc] + [2d(ab + c^2 + 4a^2d^2) + bc]k^2 - 2d^3k^4}{\alpha(k^2)}, \tag{3.14d}$$

$$\lambda_2(k^2) = -\frac{\lambda_1(k^2)}{k^2}, \tag{3.14e}$$

$$\lambda_3(k^2) = \frac{-8a^2ce + 2(ce + 2a^2dh)k^2 - dhk^4}{k^2 \alpha(k^2)}, \tag{3.14f}$$

$$\nu_1(k^2) = bg \frac{4a^2 - k^2}{\alpha(k^2)}, \tag{3.14g}$$

$$\nu_2(k^2) = \frac{1}{\lambda_A} \frac{\gamma(k^2)}{k^4(4a^2 - k^2)\alpha(k^2)}, \tag{3.14h}$$

$$\nu_3(k^2) = 2b \frac{16a^4e + 8a^2(2a^2d^2 - e)k^2 + (e - 8a^2d^2)k^4 + d^2k^6}{k^2(4a^2 - k^2)\alpha(k^2)}, \tag{3.14i}$$

where $k^2 = k_\mu k^\mu$ and

$$\alpha(k^2) = 16a^2e^2 - 4(e^2 + a^2 f)k^2 + (16a^2d^4 + f)k^4 - 4d^4k^6, \tag{3.15a}$$

$$\beta(k^2) = 16a^2e^2 - 4[e^2 + a^2 f + a^2 c(c+4ad)\lambda_a]k^2 + \{16a^2d^4 + f + [c^2 + 4ad(c-ad)]\lambda_a\}k^4 + d^2(\lambda_a - 4d^2)k^6, \tag{3.15b}$$

$$\gamma(k^2) = 64a^4e^2 - 16a^2(2e^2 + a^2 f + a^2 bg\lambda_A)k^2 + 4[e^2 + 2a^2(8a^2d^4 + f + bg\lambda_A)]k^4 - (32a^2d^4 + f + bg\lambda_A)k^6 + 4d^4k^8. \tag{3.15c}$$

In (3.14) and (3.15), we have considered $a = (2\sigma)^{-1}, b = (4\pi\tilde{\phi})^{-1}, c = \frac{\zeta e}{m_b}, d = \frac{\eta e}{m_b}$, $e = c^2 - ab$, $f = b^2 + 8d(bc + c^2 d + abd)$, $g = b + 4cd$, and $h = b + 2cd$.

In (3.11b), G is the propagator of the matter field. In the momentum space, this is given by

$$G(\vec{p}, E) = \left(E - \mu - \frac{\vec{p}^2}{2m_b}\right)^{-1}, \tag{3.16}$$

where E is the particle energy, $\vec{p}$ its ordinary momentum, and $\vec{p}^2 = p_1^2 + p_2^2$.

In (3.11c), the vector V_Λ gives the 3-point vertex of the model. In the momentum space, this reads

$$V_\Lambda = \left(1, \frac{1}{m_b}q_i, e, \frac{e}{m_b}q_j\right). \tag{3.17}$$

Finally, in (3.11c), the matrix $W^{\Lambda\Pi}$ gives the 4-point vertex. This is written as

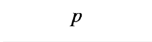
$$W^{\Lambda\Pi} = -\frac{1}{2m_b}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & e & 0 \\ 0 & 0 & 1 & 0 & 0 & e \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e & 0 & 0 & e^2 & 0 \\ 0 & 0 & e & 0 & 0 & e^2 \end{pmatrix}. \tag{3.18}$$

Next, the Feynman rules for propagators and vertices can be written:

(i) *Propagators*. We associate with the propagator of the gauge field X_Λ a wavy line

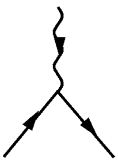
$\equiv D_{\Lambda\Pi}(k)$

and with the propagator of the fermionic matter field ψ a straight line

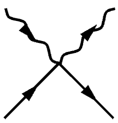


$\equiv G(\vec{p}, E).$

(ii) *Vertices*. So, the 3- and 4-point vertices of the model are



$\equiv V_\Lambda,$

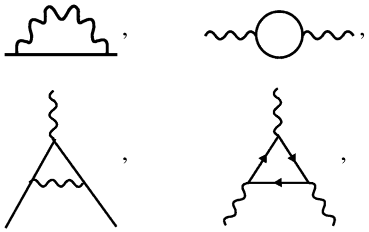


$\equiv W^{\Lambda\Pi},$

respectively.

The remaining Feynman rules are the usual ones.

Now, in the framework of the perturbative theory, a power-counting analysis shows that the primitively divergent diagrams of the model are the following:



and a graph similar to the last one with the electron arrows reversed.

We do not develop the regularization and renormalization procedures of the model in this paper. However, it can be shown that this belongs to the class of theories with only a finite number of divergent diagrams. So, the above procedures remain reduced to the ones corresponding to a superrenormalizable theory.

On the other hand, as is well-known, by adding higher-derivative terms for the gauge fields to the Lagrangian density of a model, preserving its gauge invariance, the ultraviolet behavior of the propagators corresponding to the above fields is improved and therefore the divergence of those diagrams in which these propagators appear can be removed [1, 3, 11–14, 44, 45]. Therefore, we find it interesting to consider this procedure for the present model.

We will discuss these questions in a future paper.

4 BRST Formalism

Now, we are going to apply the BRST formalism to the model.

The following brackets are all $D(F)$ ones. So, hereafter we will write these brackets without the superscription "$D(F)$".

We can write

$$[\Sigma_a(x), \Sigma_b(y)]_- = C^c_{ab}\Sigma_c(x)\delta(\vec{x}-\vec{y}), \tag{4.1}$$

with $C^c_{ab} = 0$, $a, b, c = 1, \dots, 4$.

Furthermore, let us note that the canonical Hamiltonian density, given by (2.9), can be written as follows:

$$\mathcal{H}_c = \mathcal{H}_0 - a_0\left(\frac{1}{e}\Sigma_1 + i\Sigma_2\right) - ieA_0\Sigma_2, \tag{4.2}$$

where $H_0 = \int d^2x\mathcal{H}_0$ verifies

$$[H_0, \Sigma_a(x)]_- = D^b_a\Sigma_b(x), \tag{4.3}$$

with $D^b_a = 0$, $a, b = 1, \dots, 4$.

On the other hand, due to the arbitrariness of the Lagrange multipliers ρ^a, the Hamiltonian density appearing in (2.18) can also be written in the following way:

$$\mathcal{H}_e = \mathcal{H}_0 - \rho^a\Sigma_a. \tag{4.4}$$

In the BRST formalism, the multipliers ρ_a are considered as independent dynamical field variables, associating with them the corresponding canonically conjugate momenta ξ^a. Therefore, it is fulfilled

$$[\rho_a(x), \xi^b(y)]_- = \delta_a^b \delta(\vec{x} - \vec{y}). \tag{4.5}$$

At the same time, the equations $\xi^a = 0$, $a = 1, \ldots, 4$, are imposed. Thus, the first-class constraints obtained in this way generate the gauge transformations $\rho_a \to \rho_a + u_a$ of the multipliers, in accordance with their arbitrariness.

Therefore, in this context, the independent dynamical field variables of the model are

$$A_{\mathcal{F}} = (a_\mu, A_\nu, \psi_\alpha, \psi_\beta^\dagger, \rho_a), \tag{4.6}$$

the corresponding canonically conjugate momenta are

$$P^{\mathcal{F}} = (p^\mu, P^\nu, \pi_\alpha^\dagger, \pi_\beta, \xi^a), \tag{4.7}$$

and the first-class constraints are defined by the functions

$$\Xi_A = (\Sigma_a, \xi_b). \tag{4.8}$$

In these equations, the compound indices $\mathcal{F}$ and A run over the components of the different field variables.

So, (4.1) and (4.3) take the form

$$[\Xi_A(x), \Xi_B(y)]_- = C_{AB}^C \Xi_C(x) \delta(\vec{x} - \vec{y}), \tag{4.9a}$$

$$[H_0, \Xi_A(x)]_- = D_A^B \Xi_B(x), \tag{4.9b}$$

respectively, where all the coefficients C_{AB}^C and D_A^B vanish.

The BRST-invariant Hamiltonian density is given by

$$\mathcal{H}_1 = \mathcal{H}_0 + \mathsf{P}_B D_A^B \mathsf{Q}^A = \mathcal{H}_0, \tag{4.10}$$

where Q^A are the fermionic ghost field variables (Majorana spinors) and P_A their canonically conjugate momenta. Thus, these field variables satisfy

$$[\mathsf{Q}_A(x), \mathsf{P}^B(y)]_+ = \delta_A^B \delta(\vec{x} - \vec{y}). \tag{4.11}$$

The BRST-invariant gauge-fixed Hamiltonian density is written as

$$\mathcal{H}_\chi(x) = \mathcal{H}_1(x) - \int d^2y [\chi(x), Q(y)]_+ = \mathcal{H}_0(x) - \int d^2y [\chi(x), Q(y)]_+, \tag{4.12}$$

where $\chi = \mathsf{P}_A \Upsilon^A$ is the gauge-fixing variable, with

$$\Upsilon^A = -(\rho^a, \Theta^b). \tag{4.13}$$

In (4.12), Q is the BRST generator defined as

$$Q = \Xi_A \mathsf{Q}^A + \frac{1}{2} \mathsf{P}_C C_{AB}^C \mathsf{Q}^A \mathsf{Q}^B = \Xi_A \mathsf{Q}^A. \tag{4.14}$$

We assume that the set of ghosts can be partitioned into two subsets

$$\mathsf{Q}_A = (\mathsf{q}_a, \mathsf{p}_b), \tag{4.15a}$$

$$\mathsf{P}^A = (\mathsf{p}^{\dagger a}, \mathsf{q}^{\dagger b}), \tag{4.15b}$$

where we have used the symbol "†" to point out antighosts. So, it is fulfilled

$$[\mathsf{q}_a(x), \mathsf{p}^{\dagger b}(y)]_+ = \delta_a^b \delta(\vec{x} - \vec{y}), \tag{4.16a}$$

$$[\mathsf{p}_a(x), \mathsf{q}^{\dagger b}(y)]_+ = \delta_a^b \delta(\vec{x} - \vec{y}). \tag{4.16b}$$

Consequently, $\mathcal{H}_\chi$ remains

$$\mathcal{H}_\chi(x) = \mathcal{H}_0(x) + \mathsf{p}_a^\dagger(x)\mathsf{p}^a(x) + \Sigma_a(x)\rho^a(x) + \xi_a(x)\Theta^a(x) + \mathsf{q}_a^\dagger(x)\int d^2y[\Theta^a(x), \Sigma_b(y)]_- \mathsf{q}^b(y). \tag{4.17}$$

Since $[\Theta^a(x), \Sigma_b(y)]_- = f_b^a \nabla^2 \delta(\vec{x} - \vec{y})$, where f_b^a are the elements of the matrix

$$f_b^a = \begin{pmatrix} -e & 0 & 0 & 0 \\ 1 & \frac{i}{e} & 0 & 0 \\ 0 & 0 & -\frac{2\eta e}{m_b} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.18}$$

the last term on the right-hand side of (4.17) remains $f_b^a \mathsf{q}_a^\dagger(x)\nabla^2\mathsf{q}^b(x)$.

Since the system has first- and second-class constraints, as it was shown in Sect. 2, the BRST generating functional is given by the following path integral [15, 22]:

$$Z_\chi = \int \mathbb{D}A_{\mathcal{F}}\mathbb{D}P^{\mathcal{F}}\mathbb{D}\mathsf{Q}_A\mathbb{D}\mathsf{P}^A\delta(\Gamma_I)(\det F)^{1/2}\exp\left(i\int d^3x\,\mathcal{L}_\chi\right), \tag{4.19}$$

where $\det F$ is given by (2.15) and $\mathcal{L}_\chi$ is the BRST Lagrangian density defined by

$$\mathcal{L}_\chi = \dot{A}_{\mathcal{F}}P^{\mathcal{F}} + \mathsf{P}^A\dot{\mathsf{Q}}_A - \mathcal{H}_\chi. \tag{4.20}$$

Now, we are going to prove that the expression (4.19) for the generating functional coincides with the one constructed by using the FS method, given by (3.1).

The path integrals over p_a and $\mathsf{p}^{\dagger a}$ are Gaussian and they are performed easily.

Next, we must pass to a nonrelativistic gauge [22, 54]. The steps to do so are the following: carry out the replacement $\Theta^a \to \varepsilon^{-1}\Theta^a$ in the generating functional (4.19), make a scale change in the integration variables $\xi^a \to \varepsilon\xi^a$ and $\mathsf{q}_a^\dagger \to \varepsilon\mathsf{q}_a^\dagger$, and pass to the limit $\varepsilon \to 0$ (this is possible due to the Fradkin–Vilkovisky theorem). The outcome is

$$Z_\chi = \int \mathbb{D}a_\mu\mathbb{D}p^\mu\mathbb{D}A_\nu\mathbb{D}P^\nu\mathbb{D}\psi_\alpha\mathbb{D}\pi_\alpha^\dagger\mathbb{D}\psi_\beta^\dagger\mathbb{D}\pi_\beta\mathbb{D}\rho_a\mathbb{D}\xi^a\mathbb{D}\mathsf{q}_a\mathbb{D}\mathsf{q}_b^\dagger \times\delta(\Gamma_I)(\det F)^{1/2}\exp\left(i\int d^3x\mathcal{L}'_\chi\right), \tag{4.21}$$

where

$$\mathcal{L}'_\chi = \dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger\pi - \mathcal{H}_0 - \Sigma^a\rho_a - \xi^a\Theta_a - \mathsf{q}_a^\dagger[\Theta^a, \Sigma_b]_-\mathsf{q}^b. \tag{4.22}$$

The integrations over ρ_a and ξ^a are elementary and the one corresponding to the last term of (4.22) is given by

$$\int \mathbb{D}\mathsf{q}_a \mathbb{D}\mathsf{q}^{\dagger b} \exp\left[-i\int d^3x \mathsf{q}_a^{\dagger}(x)\int d^2y [\Theta^a(x), \Sigma_b(y)]_- \mathsf{q}^b(y)\right]$$
$$= -\frac{i}{2\pi}(\det G)^{1/2}, \tag{4.23}$$

where $\det G$ is given by (2.24).

Therefore, the final result coincides with the expression (3.1) for the generating functional, as we said. Thus, we conclude that both procedures can be considered equivalent.

5 Comparison Between the Present and Original Models

Now, we are going to compare the obtained results with the ones corresponding to the original model [40].

In the present model, different from the original one, we have assumed that the real magnetic flux is attached to the particles. This is due to the existence of the topological mass term for the electromagnetic field, given by the first term on the right-hand side of (2.5a). Besides, we have considered interaction terms between the gauge fields, given by (2.5b).

On the other hand, as it can be shown, by removing the interaction terms in the Lagrangian density (2.4), we obtain a new model with the following constraint structure:

(i) The bosonic first-class constraints given by

$$\Sigma_1' = -\frac{1}{2\sigma}\varepsilon^{ij}\partial_i A_j + e\partial_i p^i - \partial_i P^i + \frac{e}{4\pi\tilde{\phi}}\varepsilon^{ij}\partial_i a_j \approx 0, \tag{5.1a}$$

$$\Sigma_2' = -\frac{i}{e}\left(\frac{1}{2\sigma}\varepsilon^{ij}\partial_i A_j + \partial_i P^i\right) + \psi^{\dagger}\pi - \psi\pi^{\dagger} \approx 0, \tag{5.1b}$$

and the ones of the same kind defined by the functions Σ_3 and Σ_4.

(ii) The fermionic second-class constraints defined by $\Omega_\alpha^{\dagger}$ and Ω_α and the new bosonic second-class constraints given by (see (2.6b))

$$\Phi_2^{0i} = p^i - \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}a_j \approx 0. \tag{5.2}$$

Now, by removing the topological mass term, we get obviously the original model. Its constraint structure is obtained directly from the one above by canceling the corresponding terms in (5.1).

On the other hand, as we noted in Sect. 3, due to the presence of the interaction terms in the Lagrangian density, it was necessary to consider a unique bosonic propagator. As we just said, by canceling these terms, we obtain a new model. As it is easy to see, for this new model, the extradiagonal blocks $L_{\mu\nu}$ of the propagator (3.12) cancel and the diagonal ones are written

$$M_{\mu\nu}(k) = \frac{1}{\lambda_a}\frac{k_\mu k_\nu}{k^4} + 2i\pi\tilde{\phi}\varepsilon_{\mu\nu\rho}\frac{k^\rho}{k^2}, \tag{5.3a}$$

$$N_{\mu\nu}(k) = g_{\mu\nu}\frac{1}{\frac{1}{\sigma^2}-k^2} + \left(\frac{1}{\lambda_A} - \frac{k^2}{\frac{1}{\sigma^2}-k^2}\right)\frac{k_\mu k_\nu}{k^4} + i\varepsilon_{\mu\nu\rho}\frac{k^\rho}{\sigma(\frac{1}{\sigma^2}-k^2)k^2}. \tag{5.3b}$$

In these equations, $M_{\mu\nu}$ and $N_{\mu\nu}$ are the CS and electromagnetic field propagators, respectively.

Next, the propagators of the original model can be obtained directly by removing the terms with topological mass in (5.3b).

As it is easy to note, the propagators (5.3) have the same ultraviolet behavior as the ones corresponding to the original model.

In inverse order, by removing first the topological mass term in (2.5a), we obtain again a new model.

In this case, the constraint structure and the bosonic propagator are obtained directly from the ones corresponding to the general model by means of that removal.

As it is easy to see, the new bosonic propagator has the same ultraviolet behavior as the one corresponding to the general model.

Next, by removing the interaction terms, we get again the original model.

On the other hand, the original model has two 3-point vertices and three 4-point ones. Furthermore, in that model, there are thirty-seven primitively divergent diagrams, twenty of them with two vertices and the rest with three ones. As it can be seen, these results are very different from the ones obtained in Sect. 3, for the present model.

Consequently, we see that both the constraint structure and the diagrammatic one, for the present model, are completely different from the ones corresponding to the original model.

Finally, as we said, for both models, the generating functional obtained from the BRST formalism is equivalent to that found by following the FS method.

6 Conclusions and Outlook

A classical nonrelativistic $U(1) \times U(1)$ gauge field model that describes the electromagnetic interaction of composite particles in $(2+1)$ dimensions has been constructed. This was done by generalizing the model proposed in [40].

Later on, the canonical quantization was realized. The model under consideration was analyzed within the framework of the Dirac algorithm.

Next, by following the FS procedure, the path integral quantization method was implemented. This enabled us to establish the Feynman rules of the model and analyze its diagrammatic structure. The model has two vertices, a 3-point one and the other a 4-point one.

Afterwards, the BRST formalism was applied to the model. The generating functional obtained from this mechanism turned out to be equivalent to that found by following the FS method, as it should be expected.

Finally, the results found were compared with the ones corresponding to the original model.

The paper was developed considering explicitly the CF case.

In future papers, we will apply the results obtained within the framework of condensed matter.

Furthermore, we will compare the results found with the ones corresponding to other models.

On the other hand, as it was mentioned in Sect. 3, we will study the perturbative development for the present model. Also, we will analyze the model obtained by adding higher-derivative terms to the Lagrangian density for the present model.

References

1. Alvarez-Gaumé, L., Labastida, J.M.F., Ramallo, A.V.: Nucl. Phys. B **334**, 103 (1990)
2. Arovas, D.P., Schrieffer, J.R., Wilczek, F., Zee, A.: Nucl. Phys. B **251**, 117 (1985)
3. Avdeev, L., Grigoryev, G., Kazakov, D.: Nucl. Phys. B **382**, 561 (1992)
4. Becchi, C., Rouet, A., Stora, R.: Phys. Lett. B **52**, 344 (1974)
5. Becchi, C., Rouet, A., Stora, R.: Ann. Phys. (N.Y.) **98**, 287 (1976)
6. Casalbuoni, R.: Nuovo Cimento A **33**, 115 (1976)
7. Casalbuoni, R.: Nuovo Cimento A **33**, 389 (1976)
8. Dirac, P.A.M.: Can. J. Math. **2**, 129 (1950)
9. Dirac, P.A.M.: Lectures on Quantum Mechanics. Yeshiva University Press, New York (1964)
10. Faddeev, L.D.: Theor. Math. Phys. **1**, 1 (1970)
11. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: Int. J. Theor. Phys. **34**, 1 (1995)
12. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: Int. J. Theor. Phys. **34**, 1037 (1995)
13. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: J. Math. Phys. **37**, 84 (1996)
14. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: Specul. Sci. Technol. **20**, 3 (1997)
15. Fradkin, E.S., Fradkina, T.E.: Phys. Lett. B **72**, 343 (1978)
16. Fradkin, E.S., Vilkovisky, G.A.: Phys. Lett. B **55**, 224 (1975)
17. Girvin, S.M.: In: Prange, R.E., Girvin, S.M. (eds.) The Quantum Hall Effect. Springer, New York (1987)
18. Girvin, S.M., MacDonald, A.H.: Phys. Rev. Lett. **58**, 1252 (1987)
19. Giuliani, G.F., Vignale, G.: Quantum Theory of the Electron Liquid. Cambridge University Press, Cambridge (2005)
20. Halperin, B.I.: Phys. E **20**, 71 (2003)
21. Halperin, B.I., Lee, P.A., Read, N.: Phys. Rev. B **47**, 7312 (1993)
22. Henneaux, M.: Phys. Rep. **126**, 1 (1985)
23. Jacak, L., Sitko, P., Wieczorek, K., Wojs, A.: Quantum Hall Systems: Braid Groups, Composite Fermions, and Fractional Charge. Oxford University Press, Oxford (2003)
24. Jackiw, R., Templeton, S.: Phys. Rev. D **23**, 2291 (1981)
25. Jain, J.K.: Phys. Rev. Lett. **63**, 199 (1989)
26. Jain, J.K.: Phys. Rev. Lett. **63**, 1223 (1989)
27. Jain, J.K.: Phys. Rev. B **40**, 8079 (1989)
28. Jain, J.K.: Phys. Rev. B **41**, 7653 (1990)
29. Jain, J.K.: In: Das Sarma, S., Pinczuk, A. (eds.) Perspectives in Quantum Hall Effects. Wiley, New York (1997)
30. Jain, J.K.: Phys. Today, April 2000, 39
31. Jain, J.K.: Phys. E **20**, 79 (2003)
32. Jain, J.K.: Composite Fermions. Cambridge University Press, Cambridge (2007)
33. Jain, J.K., Kamilla, R.K.: Int. J. Mod. Phys. B **11**, 2621 (1997)
34. Jain, J.K., Kamilla, R.K.: In: Heinonen, O. (ed.) Composite Fermions. World Scientific, Singapore (1998)
35. Jeon, G.S., Graham, K.L., Jain, J.K.: Phys. Rev. B **70**, 125316 (2004)
36. Jeon, G.S., Peterson, M.R., Jain, J.K.: Phys. Rev. B **72**, 35304 (2005)
37. Lee, D.-H., Fisher, M.P.A.: Phys. Rev. Lett. **63**, 903 (1989)
38. Lee, D.-H., Zhang, S.-C.: Phys. Rev. Lett. **66**, 1220 (1991)
39. Lopez, A., Fradkin, E.: Phys. Rev. B **44**, 5246 (1991)
40. Manavella, E.C.: Int. J. Theor. Phys. **40**, 1453 (2001)
41. Marnelius, R.: Introduction to the quantization of general gauge theories. Institute of Theoretical Physics preprint, Göteborg (1981)
42. Milovanović, M.V.: Phys. Rev. B **70**, 85307 (2004)
43. Milovanović, M.V., Stanić, I.: Phys. Rev. B **72**, 155306 (2005)
44. Nesterenko, V.V.: J. Phys. A: Math. Gen. **22**, 1673 (1989)
45. Odintsov, S.D.: Z. Phys. C **54**, 531 (1992)
46. Read, N.: Phys. Rev. Lett. **62**, 86 (1989)
47. Ryder, L.H.: Quantum Field Theory. Cambridge University Press, Cambridge (1996)
48. Senjanovic, P.: Ann. Phys. (N.Y.) **100**, 227 (1976)
49. Shahar, D., Tsui, D.C., Shayegan, M., Shimshoni, E., Sondhi, S.L.: Science **274**, 589 (1996)
50. Shimshoni, E., Sondhi, S.L., Shahar, D.: Phys. Rev. B **55**, 13730 (1997)
51. Simon, S.H., Rezayi, E.H., Milovanović, M.V.: Phys. Rev. Lett. **91**, 46803 (2003)

52. Stanić, I., Milovanović, M.V.: Phys. Rev. B **71**, 35329 (2005)
53. Stormer, H.L., Tsui, D.C., Gossard, A.C.: Rev. Mod. Phys. **71**, S298 (1999)
54. Sundermeyer, K.: Constrained Dynamics. Springer, Berlin (1982)
55. 't Hooft, G., Velman, M.: Diagramar. CERN (1973)
56. Tyutin, I.V.: Lebedev preprint FIAN 39, unpublished (1975) (in Russian)
57. Wilczek, F.: Phys. Rev. Lett. **49**, 957 (1982)
58. Zhang, S.-C.: Int. J. Mod. Phys. B **6**, 25 (1992)
59. Zhang, S.-C., Hansson, T.H., Kivelson, S.A.: Phys. Rev. Lett. **62**, 82 (1989)